Time Series Analysis in Measurement of Demand

By Anthony S. Rojko

Opinions differ among economists as to how effective statistical analyses using time series data can be in identifying factors affecting demand and in measuring their influences. Although this presentation may not materially modify these opinions, it should at least succeed in making even the most skeptical aware of some of the problems involved in analyses employing time series data. It is not the purpose of this paper to make a survey or review of previous demand studies. Instead, it concentrates on certain methodological approaches and what implications they may have in helping the analyst to measure demand. But with the many problems that face the statistical analyst in this task, perhaps good luck is what he needs most. Measurement of demand in the final analysis has no meaning unless it helps us answer some of the practical questions of economic life: Helping farmers to predict the expected price associated with given (or assumed) levels of production and consumer income, helping a Congressman to estimate the expected change in consumption if prices to farmers are raised when all other factors are left unchanged. This article is based upon a paper presented at the National Symposium on Dairy Market Development sponsored by the American Dairy Association in Chicago last November. Although the practical examples draw heavily on the dairy industry, the conclusions generally apply to all agricultural commodities. *The author gratefully acknowledges helpful suggestions from Arthur Harlow and Hyman Weingarten.*

THE BROAD PROBLEMS involved in making demand analyses, which are varied and many in number, are discussed under the following headings: (1) Some Necessary Ingredients in Demand Analysis; (2) Structural vs. Predictive Relations; (3) Simple vs. Complicated Methods; (4) Allowing for Changes in Structure; (5) Short- and Long-run Estimates of Demand; (6) Avoiding Nonsense Correlations Through Graphic Analysis and (7) How Many Variations Should We Try?

Some Necessary Ingredients in Demand Analysis

Assuming that an economic relation exists between the variables, what further conditions must be satisfied before reasonable statistical results can be obtained? Knowledge of these limitations is essential—they may influence the degree of refinement that is possible in demand analysis.

(1) As statistical analyses measure change, the first requirement is that variability in the data be sufficient to permit observation of the effect of change in one variable on other variables. For some commodities such as meat, the year-to-year variation in consumption may be substantial. Consumption of beef in the last decade varied from 56 to 85 pounds per person. The largest annual change was 15 pounds or about 25 percent. In only 2 years was the annual change less than 2 percent. In contrast, time series of dairy statistics frequently show little year-to-year change. Nonfarm per capita consumption of fluid milk and cream ranged between 329 and 333 pounds for the period 1948–54. Except for one year, all year-to-year changes were 1 to 2 pounds per capita. In the one exceptional year, consumption dropped 3 pounds, but even this dip was less than 1 percent. Per capita consumption of manufactured dairy products tends to vary more, but even here variation is not great. Per capita consumption

Reprinted by the Economic Research Service from AGRICULTURAL ECONOMICS RESEARCH, Vol. XIII, No. 2, April 1961

ERS-1 APRIL 1961

of American cheese, for example, has ranged between 5.1 to 5.5 pounds since World War II, and the greatest change in any year was 0.4 of a pound, or about 8 percent.

To get statistical results with confidence, the data on which results are based should have greater variability than the error associated with the data. In some of our consumption data for short periods, the year-to-year changes in the data are probably less than the error associated with the data. It should not be surprising, therefore, that hoped-for results in estimating demand coefficients, using postwar data alone, have been difficult to obtain.

(2) A second condition for ideal application of price analysis to time series data is that no structural change shall have taken place during the period studied. If change has taken place, may it be allowed for by statistical means? As shown elsewhere in this paper, when structural changes do occur they may be serious enough to put severe limitations on the job of running an analysis based on any considerable period of time. Yet a certain minimum number of observations is necessary if any confidence is to be placed in the coefficients.

(3) Another statistical requirement is that the intercorrelation among the explanatory variables be at a minimum. If intercorrelation is high, it not only reduces the statistical significance of the demand coefficients but also affects the size of the coefficients. As shown by Fox and Cooney (*10*, p. 4),[1] the sign of the regression coefficient may change with high intercorrelation. Thus, we may have a regression analysis which explains a large percentage of the total variation in the dependent variable. Yet, because of high intercorrelation, the individual coefficients associated with the explanatory variables may be useless.

In demand analysis pertaining to dairy products, we encounter several areas in which this problem affects statistical measurement. It is difficult, for example, to measure the substitution effect between competing dairy products using time series analysis, simply because prices of these competing products are highly correlated over time. Prices of fluid skim milk cannot get out of line with prices of fluid whole milk, or with prices of evaporated milk, as the values of all are derived from the same raw material—milk. Other intercorrelation problems result from similar trends in two or more variables. Since World War II, the steady rise in consumption of broilers has been the result of a downtrend in prices reflecting production efficiencies and an uptrend in consumer incomes. We have no statistical means of isolating these joint trends.

(4) A fourth ingredient is that no serial correlation exists among the residuals. Serial correlation is present in most residuals computed from demand analyses. This effect can sometimes be minimized by using first differences (Foote *7*, pp. 30–32 and Cochrane and Orcutt *5*, pp. 54–55) or by adding the lagged dependent variable as an additional explanatory variable in the analysis (Nerlove and Addison *27*, pp. 877–879).

(5) A fifth requirement is concerned with the specification problem. Of the several specification errors, the one with which we are mainly concerned is the one that occurs when some of the conditions specified in the economic model are not fulfilled in our estimating procedure. Specifically, the method of estimation requires that certain conditions must be met in the model to get estimates with desirable properties. The model, in turn, specifies the form of the structural equation and the restrictions imposed upon the unknown parameters. As an example, least-squares regression analysis stipulates that the covariance, or the correlation between the residual and the explanatory variables, be zero (Wold and Faxer *38*). If results do not indicate this, the estimating method does not meet the specification of the problem. An incorrect economic theory is not a specification error—it is just wrong theory. Errors of observation in the data and the use of nonrepresentative series to reflect certain price or quantity changes also illustrate kinds of specification error. The problem of selecting representative series in any demand analysis can be perplexing—just plain rough. For example, we do not have a representative price for fluid whole milk that reflects fully some of the changes that have occurred in purchasing habits. The price series [2] of the last few

[1] Italic numbers in parentheses refer to Literature Cited, page. 52.

[2] MacPherson and Smith (*21*, pp. 5–6) state: "Average selling prices for milk packaged in types and sizes of containers used in households have declined over the 4-year period of April–June 1956 to April–June 1959 . . . . Listed selling prices for a selected group of fluid milk

years may have a slight upward bias since it does not fully reflect the shift to gallon purchases of milk at lower per unit (quart) costs.

Structural vs. Predictive Relations

Too frequently, people want a single and unique answer to a problem. They are not interested in *several* demand coefficients. They want a *single* demand elasticity which they can firmly fix in their minds for use in answering all their problems. But it is possible to obtain several different demand coefficients, depending upon the number of variables in a regression equation or upon the method of statistical fit. Users of results need to know that each coefficient may have a special use yet may suffer serious limitations in other uses. Take the case of the question: What changes may be expected in consumption of butter if incomes are to be raised by, say, 10 percent in the next year? Can we give a single answer? Should we give an income elasticity figure from a structural demand relation that is consistent with the Marshallian demand curve?[3] But will all other things remain constant—the *ceteris paribus* assumption under the Marshallian demand curve? Initially, at least theoretically, an increase in income results in a shift in the demand curve for butter by the amount specified by the income elasticity coefficient. But with a given supply and an increase in demand, the price of butter would rise. In effect, this would reduce the increase due to income and, at the same time, encourage greater supplies. But income also increases the demand for fluid milk and other dairy products. Results from statistical analyses indicate that if the total supply of milk cannot be increased to meet the increase in demand for total dairy products, consumption of butter will actually decrease even though the demand for it has increased. We have witnessed, of course, how the utilization pattern shifts in favor of fluid milk at the expense of butter when milk is in short supply. In short, the demand elasticity coefficient, based on *ceteris paribus* conditions, is useless unless we want to trace out the series of adjustments by an iterative process. A more useful coefficient is one that already takes into account all these intermediary adjustments. Thus, the kind of coefficient needed is what Buse (*3*) calls the total elasticity from the total demand response curve.

On the other hand, if prices of butter are at support levels and the Commodity Credit Corporation has stocks of butter, the increase in demand for butter, specified by the Marshallian demand elasticity, would be reflected in a like increase in consumption. In this case we get no simultaneous price effect from the supply side, at least no material effect, until CCC stocks are almost exhausted.

Can this dilemma be resolved? Perhaps not. Which answer should our time series analysis provide? We can minimize the confusion if we distinguish between structural and predictive relations. These two relations are not one and the same. The first is concerned with relations within the Marshallian concept of supply and demand curves. Predictive relations are those which are designed to give an estimate of the variable in question, such as price or consumption.[4]

Back in 1927, Elmer Working (*39*) recognized the difference between predictive relations and true structural demand relations.[5] In 1943, in a pioneer work, Haavelmo (*13*) showed statistically why the coefficients associated with the predictive relation were not the same as those in the structural relation, even though both relations in-

(Footnote 2 continued from page 38.)

distributors indicate that this decline has occurred in spite of the fact that prices for milk in individual types and sizes of containers have increased. The paradox of average prices declining while individual prices increase can be accounted for by two changes: First, and most important, a shift to larger containers at lower per quart prices; second, a shift away from higher priced varieties of milk . . . ."

[3] Structural relations are those that define the process by which a set of economic variables are believed to be generated.

[4] Statistically, the predictive relation can be looked upon as an estimator or general formula to which given or known observations can be applied to compute an estimate. In the field of statistics, we have a wide variety of estimators—unbiased estimators, consistent estimators, efficient estimators and sufficient estimators; minimum variance estimators, minimum x^2 estimators, minimum root mean square error estimators and maximum likelihood estimators; Baye's estimators, fiducial estimators, and least squares estimators; closest estimators and minimum confidence interval estimators, and, among others, believe it or not, "best" estimators.

[5] A similar line of reasoning is followed by Koopmans (*16*, pp. 27–35) and Foote (*6*).

volved the same variables. In fact, Haavelmo's article was the catalyst for the simultaneous equations work that followed its publication. It has inspired many econometricians to search for methods of quantifying the underlying basic structural relationships, such as the "true" demand and supply curves of economic theory—so much so that some analysts soon came to believe that least squares as a method of quantifying economic behavior was "old fashioned" and "outmoded." Others looked upon the simultaneous equations approach as a mystic manipulation, too difficult to comprehend. But of course neither idea was correct; each of these tools has a place in our kit.

The least squares approach is useful in showing what normal average relationships exist between sets of variables. Besides, it appears that some of the same factors that help or hinder reasonable least squares results also do so in the more refined methods, such as limited information method and two-stage least squares method. The important thing is to recognize that the simultaneous equations approach may be needed to determine statistically the coefficients in the structural relations. These, in turn, may be necessary to establish algebraically the predictive or estimating relations.[6] On the other hand, the least squares equation may be the most economical and efficient way of obtaining these predictive relations. This is the other aspect of the Haavelmo paper, the one that apparently has been ignored.[7] Foote and Waugh (*8*), Hildreth (*14*), Christ (*4*), and Klein (*15*) each review the merits of simultaneous and single equations. They stress that both methods of analysis are essential.[8] On the other hand, Wold (*37*) questions the presence of simultaneity in economic relationships and suggests that recursive relations fitted by least squares are more appropriate.

Closely associated with predictive relations are those formulations that can be used for evaluating policy programs. In many respects, such appraisals are only extensions of forecasting with changed structures. Difficulties arise when the hypotheses, assumptions, and objectives involved in quantifying economic relations differ statistically from those required for use of statistical results in program appraisal. In fitting economic relations, the important considerations are the nature and the availability of data and the adequacy of the statistical method employed.

These considerations often affect the kind of formulation fitted. But in using econometric results for program appraisal, answers are sometimes obtained only from special formulations which may not meet the more rigid requirements specified in fitting procedures such as availability of data. Also, we seldom have a unique statistical formulation using a single source of data in which all important coefficients are statistically significant. As a result, several formulations often are modified so that information from several sources may be pooled into a single formulation, which is then used for appraising programs. We need more information to determine whether the use of a system that incorporates results from several analyses gives better estimates than those obtained directly from fitted regressions. There is no question but that, for the period of fit, estimates from the fitted system probably are better. But since appraisal involves change in structure, there is also no question but that a known "poor" coefficient may cause considerable difficulty. Thus it is well to know how to integrate and use all the known information.

Simple vs. Complicated Models

Let us examine some of the available statistical methods for measuring demand relationships. These methods may be simple, but they *can* also be extremely complex; it may even make sense to classify estimating methods according to *degree*

[6] In econometric literature, these equations are usually referred to as "reduced form equations." Reduced form equations are equations that result when each endogenous variable in a system of equations is written as a linear function of all of the predetermined variables in the system. Depending on the circumstances, they may be (1) algebraically derived from the structural coefficients or (2) fitted by least squares.

[7] For example, see a forthcoming article by Waugh (*36*).

[8] Some emphasize the importance of obtaining "true" structural coefficients while others stress the importance of obtaining the "best" forecast. Contrary to prevailing notions, Liu (*20*) suggests that the complexity of modern economic society makes it much more likely that the true structural relationship is *under-* rather than *over-*identified. Since simultaneous equations methods cannot be used in *under*identified models, he suggests use of relationship including all important variables fitted by least squares.

(Footnote 8 continued.)

of complexity. In this way, the kind of results that are obtained from each method can be compared with the amount of effort expended. Of course, other classifications are possible. The methods may be grouped into the single equation vs. the simultaneous equations approach. Much of the statistical measurement of demand from time series analysis falls into these two broad groups. Graphic analysis and least squares regression analysis fall in the single equation group. Listed in the approximate order of their complexity in the second group are: The reduced-form method, the two-stage least squares method, the limited information method, and the maximum likelihood full information method.

Some statistical methods lie somewhere between these two broad classifications and tend to be used less frequently. In this area are such methods as analysis of principal components, discriminant analysis, canonical correlations or regression between sets of variables, weighted regressions, distributed lags, indifference curve approach, and so on.[9] The purpose of this paper is not to discuss these methods in detail but rather to indicate broadly some of the aspects that should be considered before using any single method.

We are often under the illusion that we obtain poor results because we fail to include all the relevant information. It is true that the addition of another explanatory variable in a regression analysis may increase the coefficient of multiple determination (R^2). Since a system of equations brings to bear more information on the problem, is it safe to assume also that the results are also improved? Bigness in model construction does not necessarily mean better results.

Why do we need complex models? When several variables are jointly determined, several equations may be required to take into account this joint interrelationship. How many equations do we need? This depends upon the importance of some of these interrelationships. To account for the interrelationships among dairy products and their competing products with emphasis on competition between butter and margarine, Ladd (*18*, p. 646) developed a model containing 63 relations. This model has 13 demand equations in all. Of these only 6 are for dairy products, 6 are for competing products, and 1 is for the total demand for table fats. The other 50 equations include retail-supply, processor-supply and inventory-demand equations, production equations for 5 dairy products, equations for exports of evaporated milk and for imports of cheese, domestic shortening production equations, and margarine and shortening-ingredient price-index equations. No equations appear for the supply of milk on the farm which is presumed to be given in any year. Lifting this restriction would, of course, increase the number of equations.

Statistical methods of estimating sets of simultaneous equations are needed to estimate the "true theoretical relations," for example, the "true demand curve" for fluid milk or butter. Statistical results obtained from such systems of equations are valuable in helping us to understand the theory of interrelated markets for milk and dairy products at the different marketing levels. The Ladd study is an example of how such an integrated model helps one to understand better the interrelationships between the butter and margarine market, including the effect of certain institutional variables. The work that I have done using the simultaneous-equations approach also helps to explain why, under certain circumstances, simpler single-equation regressions do not give the kind of coefficients expected in Marshallian demand relations (*31*, p. 71). Estimation of the coefficients in the demand relation for butter is a good example.

Traditionally, butter had been considered as a buffer for sudden shifts in the supply position because of weather or other unexpected circumstances. Undoubtedly, this has reflected, in part, the general availability of equipment for making butter as opposed to that for making cheese and other manufactured products. Important also are the ease with which butter can be stored and its place in the dairy economy as an outlet for milk. As a result, to estimate consumption of butter, any analysis must take into account the supply of total milk and demand for other dairy products. As those who are familiar with the dairy industry know, consumption of fluid milk is affected little by immediate shifts in supply of total milk or the

[9] These methods are frequently overlooked in most books on econometrics. Each of these uses some form of regression analysis. Tintner (*32*) presents a good account of the first four methods and includes others not listed above. Nerlove (*26*) discusses the use of distributed lags in the measurement of demand for agricultural commodities. For an application of indifference curve approach to measurement of substitution in demand, see Waugh (*33*) and Meinken, Rojko, and King (*23*).

demand for competing dairy products. And, of course, low demand elasticity for fluid milk also tends in the very short run to insulate consumption of fluid milk from these factors. For this reason, as shown later in this paper, reasonable results may be obtained for the fluid sector using the single equation regression approach. Single equation regression analysis also suffices for butter and manufactured products when manufacturing milk prices are at support levels.

One of the primary advantages of formulating a complete model is that it provides a systematic way of taking into account all relevant information that may influence the estimate of one of the dependent (endogenous) variables. This can prove to be an important function, as it helps to suggest areas in which least squares regression analysis is sufficient. For example, results from a two-equation dairy model for milk at the farm level fitted by both the least squares and the limited information methods indicated that both methods gave approximately the same coefficients in the demand equation for total milk (*30*, p. 337). This indicated that a least squares fit was satisfactory for the demand equation. Frequently, by formulating a complete model and using our knowledge of the industry, we can select the relevant variables needed in an estimating equation that can be fitted by least squares. These equations may be used for estimating the dependent (endogenous) variables, and the total model need not be fitted by the limited information or other complex methods. In summary, the important question is, Does the statistical equation that is used to make the estimate reflect all the relevant information necessary to explain the economic behavior of the variable in question? There are several ways of incorporating this information in our estimating equation.

But enlarging the model to increase the amount of information that can be used in making estimates also brings certain disadvantages. One is related to the need for more rigid assumptions as the size of the model increases. How many analysts ever stop to question why most complex systems of equations are always fitted using linear relationships? Such models usually have additive identities. In a dairy model, the sum of the individual demands for fluid milk, cream, butter, cheese, powdered milk, and so on must equal the total demand for milk. Also, prices for dairy products at different marketing levels theoretically differ by the differences in the marketing services performed for each commodity, differences in the densities of each dairy product since prices are usually quoted on the basis of product weight, and differences in the quality of milk used in making the product. Demand and price relations must be in linear form to permit these identities. But demand relations might be multiplicative (curvilinear). If so, a regression based on data in logarithms would give a better fit. Sometimes it is possible to have semilogarithmic relationships, provided the additive variables involved can be expressed as actuals. I have seen several models in which the linear restriction had an important effect on the kind of results obtained (*9*, p. 35).[10] For similar reasons also, the same format is frequently followed for all equations in the model. That is, the data are either in actuals or in logs, or they are run as first differences of actuals or logarithms. Such uniformity is not essential in all instances. In fact, there is reason for some modifications.

Another difficulty in working with a large model is pinpointing its statistical weaknesses. In working with a single regression, several combinations of variables that appear to be consistent with theory are tried, and they are either accepted or rejected on the basis of statistical significance and intuitive judgment. It is easy to decide whether a particular variable will help or hinder results.

In working with a large model, the analyst too frequently evaluates its "goodness" by asking how many coefficients are significant, and too often he is happy when three-fourths of them meet the standard statistical test of significance. One reason for this is that, in the case of the limited information method, it is difficult to evaluate which explanatory variable is responsible for a "poor" coefficient, since all structural coefficients are jointly determined. In one of my models, I had expected the addition of the retail price of meats to affect the demand equation for cheese. But I

[10] Foote and Weingarten, in using Meinken's wheat model (*22*, pp. 36–50) to demonstrate the use of research results in analyzing alternative programs, found that it was necessary to substitute a curvilinear relationship for the feed demand for wheat in place of the fitted linear relationship. The fitted linear relationship was inadequate because, when the price of wheat approaches the price of corn, use of wheat for feed increases rapidly and by more than the quantity suggested by the linear relationship.

was surprised to find that it affected the coefficients in the demand equation for butter considerably more. It may be somewhat easier to trace out the effects in the two-stage least squares method. But if by trial and error we select only those explanatory variables that appear to give the "best" structural coefficients, we may be deluding ourselves into believing that we have the joint determination implicit in our original system of equations.

At this point, we might add that some of the difficulties in evaluating the results stem from the intercorrelation problem. In a single regression analysis, it is fairly easy to see the influence of intercorrelation on the coefficients obtained (*10*). When several equations are involved, however, we do not have a method for ascertaining the effect of high intercorrelation among the explanatory (predetermined) variables on the structural coefficient. Our experience is that these coefficients are affected and that high intercorrelation among the predetermined variables tends to contribute to larger standard errors in the structural coefficients. Some of our results, however, suggest that high intercorrelation among the predetermined variables apparently does not affect the predicting value of the estimating (reduced-form) equations, provided the economic variables stay close to their range of values included in the original analysis (*11*, p. 92). More work needs to be done in this area.

Whether the statistical model is simple or complex will depend also upon the methods used to measure substitution in demand. The three empirical measures of demand interrelationships are (1) direct and cross elasticities derived from statistical demand equations, (2) elasticity of substitution derived from price ratios and consumption ratios, and (3) partial indifference surfaces derived from demand coefficients and an assumed monotonic function of utility.[11]

In general, the research analyst will wish to obtain direct and cross price elasticities from the demand equation by the regression approach, as this method provides the greatest amount of information. Either single equation regressions or systems of equations may be used, depending upon the nature of the interrelationships between the competing products. If intercorrelation is high among the prices of substitutes, the demand function might specify these prices as price ratios. Quantities may also be expressed as ratios in the analysis. But empirical elasticities of substitution obtained by relating price ratios and consumption ratios tell us little about the "ease of substitution" or degree of competitiveness between the goods.

The complexity of the model is also affected by the way in which a static model is converted into a dynamic one. The simplest way to achieve such a conversion is to include the dependent (consumption) variable as a lagged explanatory variable in the regression. The lagged (consumption) variable in this formulation reflects past influences of prices, incomes and other factors, including customary levels of consumption. In this connection, additional discussion appears later in this paper.

Allowing for Changes in Structure

As stated earlier, time series analysis assumes that no changes in structure have occurred during the period of analysis; that if such structural changes have occurred, they can be allowed for statistically in the regression analysis. Changes that take place gradually over time and for which we have no specific explanatory variable are usually allowed for by the time variable. In this connection, some analysts may not fully realize that the addition of the time variable imposes certain restrictions on the kind of trend that results. It makes a difference, for example, whether the time variable is in actuals, in logarithms, or in some other form (7, pp. 39–43). Many regression analyses have been run using first differences of logarithms with a constant or "a" value obtained to measure the trend in the dependent variable over time. Most analysts may not be aware of the fact that this formulation only permits a trend that is increasing at an increasing rate. Intuitively, the formulation that permits the opposite may be desired. This deficiency may be corrected by taking first differences of logarithms in a demand relation which explicitly includes time as an explanatory variable.

If there has been a once-and-for-all change in structure (level of consumption), the use of a 0–1 variable may be satisfactory. To illustrate, the Special School Milk Act, passed in 1954, has resulted in a higher level of consumption of fluid

[11] For a study relating these three approaches—advantages and disadvantages of each—see Kenneth W. Meinken, Anthony S. Rojko, and Gordon A. King (*23*).

milk in a magnitude of about 7 pounds. How do we allow for this in a statistical analysis? If no substitution occurs between this milk and milk bought from commercial channels, an analysis can be run by subtracting out milk consumed under the special milk program. That is, consumption variables should represent only commercial takings. But we may not want to assume that no substitution has occurred. In this instance, the analysis should have an additional variable which designates the years through 1954 with a value of 0 and the years beginning with 1955 with a value of 1. A comparison of the results from two demand regressions for fluid whole milk for the period 1924–59 indicates how effectively the 0–1 variable can be used to reflect changes in structure. (Regressions for demand for fluid milk are given at the end of discussion, beginning with page 51.)

At this point, it may be well to add that preliminary graphic analysis can be useful in depicting changes in structure over time, as the section on graphic analysis in this paper attests.

Another way to handle changes in structure is to break the period into subperiods during which no change in structure occurred. This was done, for example, to determine the changing relationship between cheese and meat consumption over time. Analyses for the 1920's suggested that price of meat had no influence on consumption of cheese, while the analyses in the 1930's indicated the beginning of some influence. Postwar analyses, however, suggest that the price of meat is an important consideration influencing consumption of cheese. Also, although some margarine was consumed in the 1920's and 1930's, the price of margarine did not appear to influence the consumption of butter. But this is not true in the postwar period. In such instances, if the subperiods are sufficiently long, separate regressions can be run for each period and the results from each compared. But we cannot logically combine or run the analyses for the total period because the influence of the factor (price of margarine), which was relevant only during part of the period, is averaged for the whole period. This, of course, gives a meaningless coefficient. One possible way to use a single analysis for the total period would be to leave out the price of margarine and to use a 0–1 variable instead. Then the residuals from this analysis might be correlated with the price of margarine for those years in which the price of margarine could be expected to be an influence.

Short- and Long-Run Estimates of Demand

The literature abounds in confusion with regard to the definition of the length of run. Some of the confusion undoubtedly stems from the fact that the real world is a curious mixture of both short- and long-run adjustment. The difficulty occurs when we attempt to delineate how much of the current level of demand results from adjustments in the short-run or the long-run. What are some of the methods that analysts can use to measure these separate influences?

One method is to relate the period of observation used in the analysis to the length of run. In this instance, length of run depends on whether we use monthly, quarterly, annual, biennial, or longer periods of time as our time period for each observation in the time series analysis. This method assumes that the factors that affect consumption can be grouped according to length of time required for adjustment. Specifically, the quarterly analysis would measure the influence of certain factors while the annual analysis would measure only the influence of other factors. Such analyses can be rather informative. For one, quarterly analyses might depict differences in seasonal demand, such as the stronger demand for ice cream in summer than in winter. These differences presumably would cancel out when the analysis used annual data. One may analyze quarterly data in one of two ways. If the period of analysis is long enough, there are advantages to running each quarter separately, then comparing their results. One may also use the 0–1 variable concept in which all the quarters are included in the single analysis. In this instance, we add additional variables which take on the value of 0 or 1, depending on whether the period of observations includes the quarter.

The 0–1 variable is a useful tool when data are available for only a relatively short period. To illustrate, let us look at some quarterly regressions based on 13 observations using Market Research Corporation of America consumer panel data published by the Agricultural Marketing Service. The regressions based on data in logarithms for fluid whole milk are

$$X_1 = -1.065 + \underset{(1.37)}{.27X_2} + \underset{(1.33)}{.46X_3} + \underset{(.45)}{48X_4}$$

$$X_1 = 2.867 - 2.63X_2 - .20X_3 + .67X_4 - .045X_5$$
$$\quad\;(.64)\quad(.45)\quad(.16)\quad(.006)$$
$$- .038X_6 + .005X_7$$
$$(.005)\quad(.004)$$

in which X_1 is per capita purchases of fluid whole milk, X_2 the prices paid for fluid whole milk, X_3 the price of fluid skim milk, X_4 per capita disposable income, and X_5, X_6 and X_7 are for the second, third and fourth quarters, respectively, using the 0–1 concept. All economic values were deflated by the Consumer Price Index. The numbers in brackets are the standard errors of the regression coefficients. None of the coefficients in the first analysis are statistically significant. The coefficient of determination (R^2) was increased from .27 to .95 by allowing for seasonal differences. The price coefficient now has the correct sign although a demand elasticity of −2.63 is much too high. As expected, the analysis shows that consumption in spring and summer is lower than consumption in winter by about 10 percent. Consumption in fall is 1 percent higher, or about the same as in winter. These results should be used with caution because they are based on a relatively short period of time. The regressions are inserted to illustrate the effectiveness of the 0–1 variable.

In many instances, it is impossible to correlate the period of actual adjustment with some time period. This is true because adjustments are continuously taking place. Prices of agricultural products change more often than once a year. A statistical analysis that arbitrarily specifies periods, such as a year, measures only the average relationship between the variable involved. It is not surprising, therefore, that coefficients from an annual analysis including 10 years of observations differ from those based on longer periods such as 20 or 30 years. Factors that exhibit cyclical behavior are particularly affected.

Some analysts have defined short- and long-run coefficients in this context. They have run a regression for the total period and then separate regressions for subperiods. An analysis by decades might be run for the 1920's, the 1930's, and so on. A single analysis would also be run for the total period 1920–60. In each instance, the period of observation would be a year, yet the coefficients from the subregressions would reflect short-run factors, whereas the coefficients in the longer period analysis would tend to reflect long-run changes. Relative prices and the transient component of income could be expected to be of greater significance in the shorter analysis, while in the longer analysis, level of income would reflect longer-run changes.

Still another method to delineate between short- and long-run elasticities has been given increasing attention within the last decade. This method implies that we are continually making short-run adjustments which are superimposed upon some underlying long-run adjustment that the consumer seeks to attain. Implicit in the method is the fact that it takes several periods to make the adjustment following a given change in one factor while all other factors remain constant.

Two decades ago, Mighell and Allen (*24*) recognized the difference between instantaneous and normal adjustment to price changes. Elmer Working (*40*) made the first serious attempt to measure the difference between the short- and long-run elasticities of demand in this context. His approach consists essentially of using different moving averages of quantity and income to explain the level of current price. The length of run implied in the coefficient is directly related to the period covered by the average used. Some differences followed as to the interpretation that should be given to the demand coefficients obtained by Working (*2, 17, 29, 12*). Ladd and Tedford (*19*) suggest a reasonable interpretation. They demonstrate that the Working method is a special case of a more generalized method. The method assumes that the current level of consumption is the result of past decisions on the part of consumers, as well as recent adjustment to the most recent change in price, income, or some other causal factor. Current price, price the year before, the price in the year before that, and on into the past, each had some influence on the present level of consumption. The more distant in the past, the less influence price exerts. If all past prices and incomes are included in the same analysis, the resulting high intercorrelation between prices and income over time poses serious statistical problems. For this reason, some analysts have used averages of past prices or incomes. Others have avoided this intercorrelation problem by using lagged consumption to reflect the influence of the past on current levels of consumption.

Nerlove (*25*, *26*, *27*) embarked upon a new approach to the estimation of short- and long-run elasticities of demand. He applied to the field of agriculture certain concepts of distributed lags which were known to econometricians but had somehow escaped the notice of agricultural economists. Nerlove's approach has several points in common with Working's method. Although Working did not specify a long-run function, as did Nerlove, such a function is implicit in the Working method. Nerlove's approach assumes that there is some long-run equilibrium quantity (consumption) which consumers attempt to achieve by continually making adjustments in the short-run in moving toward this long-run equilibrium. Since prices and incomes do not remain still long enough for complete adjustment to this equilibrium, we cannot observe the long-run equilibrium position statistically because it does not exist. As a result, the long-run demand function cannot be estimated directly. Nerlove gets around this difficulty neatly by defining an adjustment equation which, combined with the long-run demand function, gives an estimating equation in terms of observable variables. Equations (2) and (4) on pages 51 and 52 are examples of such estimating equations. Thus, from the information in the estimating equation which he fits by least squares and the adjustment equation, Nerlove computes algebraically the long-run demand elasticities. One of the variables in the estimating equations is lagged consumption. The Nerlove approach uses lagged consumption to reflect the influence of past prices and past incomes, while the Working method uses moving averages of past values. One of the real advantages of the Nerlove approach is that it reduces serial correlation in the residuals of the estimating equations.

As indicated by Brandow (*1*), the long-run elasticities so obtained can be affected by specification errors. Why can this be so? The relation between short- and the long-run elasticities is materially affected by the coefficient associated with the lagged consumption variable in the estimating equation. Implicit in the method is that the lagged consumption variable reflects *only* the past influence on current consumption of past prices, incomes, and other factors specified in the model. But past consumption also may reflect other factors. For example, Nerlove's short- and long-run elasticities may be obtained from some of the demand regressions for fluid milk on pages 51 and 52. Using equation (2), we can obtain a short-run price elasticity coefficient of $-.32$ and a long-run price elasticity of $-.70$. But if we use equation (4), the short-run price elasticity becomes $-.37$ while the corresponding long-run elasticity becomes $-.56$. These results indicate that specification errors can affect the estimates of long-run elasticities. For these same equations, the relationship between the short- and long-run elasticities for income would be the same as those shown for price.

As expected from economic theory, the long-run demand elasticities for fluid whole milk obtained by the Nerlove method are greater than elasticities obtained from regression analysis in the typical demand equation using annual data and longer periods of time, such as 20 or 30 years. In the latter, the corresponding price elasticities were $-.62$ and $-.44$ (from equations (1) and (2), respectively). The Nerlove long-run elasticities are those that give the total adjustment that would occur in consumption following a single change in price or income over a period of several years. These coefficients also assume that all other factors remain the same. But do all other factors remain the same? One should not confuse these long-run elasticities with those needed for making projections 10 or 20 years ahead. It may be that the average relationship obtained from a regression analysis based on a longer period of analysis, say 20 or 30 years, provides the more reasonable answer for use in these long-run projections. It may be that the real world is but a series of short-run adjustments and that the total adjustment implied by the long-run demand elasticities is never obtained. It may be that, in the long long-run, the elasticities for certain items, such as dairy products, are rather low. Some signs indicate just such a position.

Analysis of survey or cross-section data indicates that the income elasticities are lower for people in the high-income bracket than for those in the low-income bracket. One of the marks of a progressive economy is that in addition to the rise in the level of real income, income disparities become less pronounced. These two elements would tend toward the lowering of demand elasticities over time. This would definitely be true for milk at the farm level. However, the increase in the demand for marketing services, which apparently

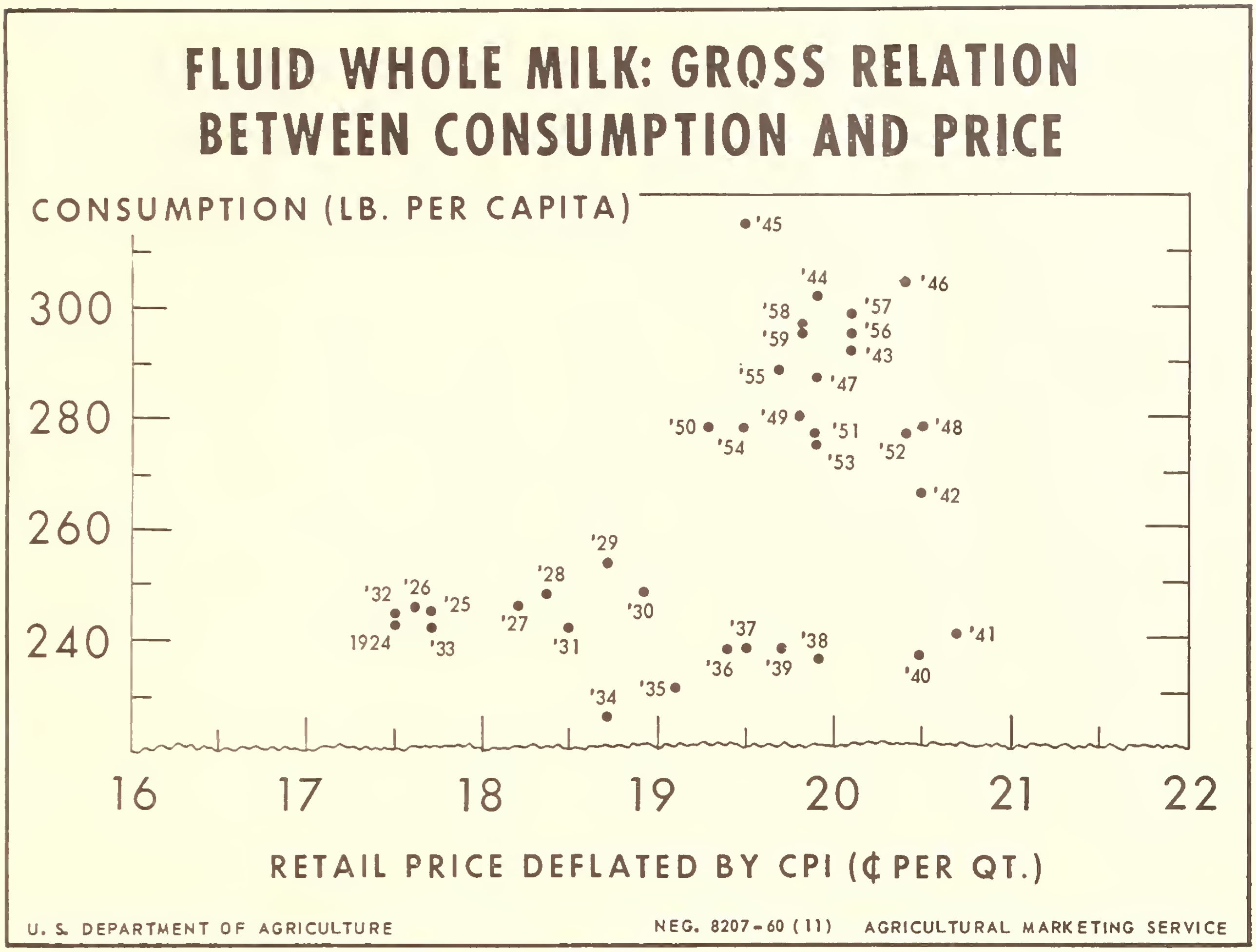

Figure 1.

is also associated with a rising economy, would tend to maintain the demand elasticities at the consumer level. But this area is beyond the scope of this paper, even though it poses some interesting avenues to adventure.

Whatever interpretation of the long-run demand elasticities obtained by the Nerlove method may be made, it is desirable to stress the fact that the estimating equation developed in his method is useful in forecasting consumption a year or two in advance.

The concept of distributed lags has recently been applied to time series data in measuring the effect of expenditures for advertising and promotion on the demand for farm products. Nerlove and Waugh (*28*, *35*) used this concept in the analysis of returns to orange growers from producer-financed advertising during the last 50 years. In their study, a method was developed to measure the long-run rate of return on advertising expenditures that could then be equated to returns from other forms of investment.

Avoiding Nonsense Correlations Through Graphic Analysis

Some agricultural economists and statisticians have become so intrigued with new mathematical methods and computing techniques that they have neglected a powerful simple tool—graphic analysis.[12] The greatest value of graphics in research is in making a quick preliminary analysis to determine the relevant variables and the form of the relationships among these variables. In addition, graphic analysis can be of material value in pinpointing changes in structure over time. Further-

[12] For the many uses of graphic analysis in agricultural economics, see Waugh (*34*).

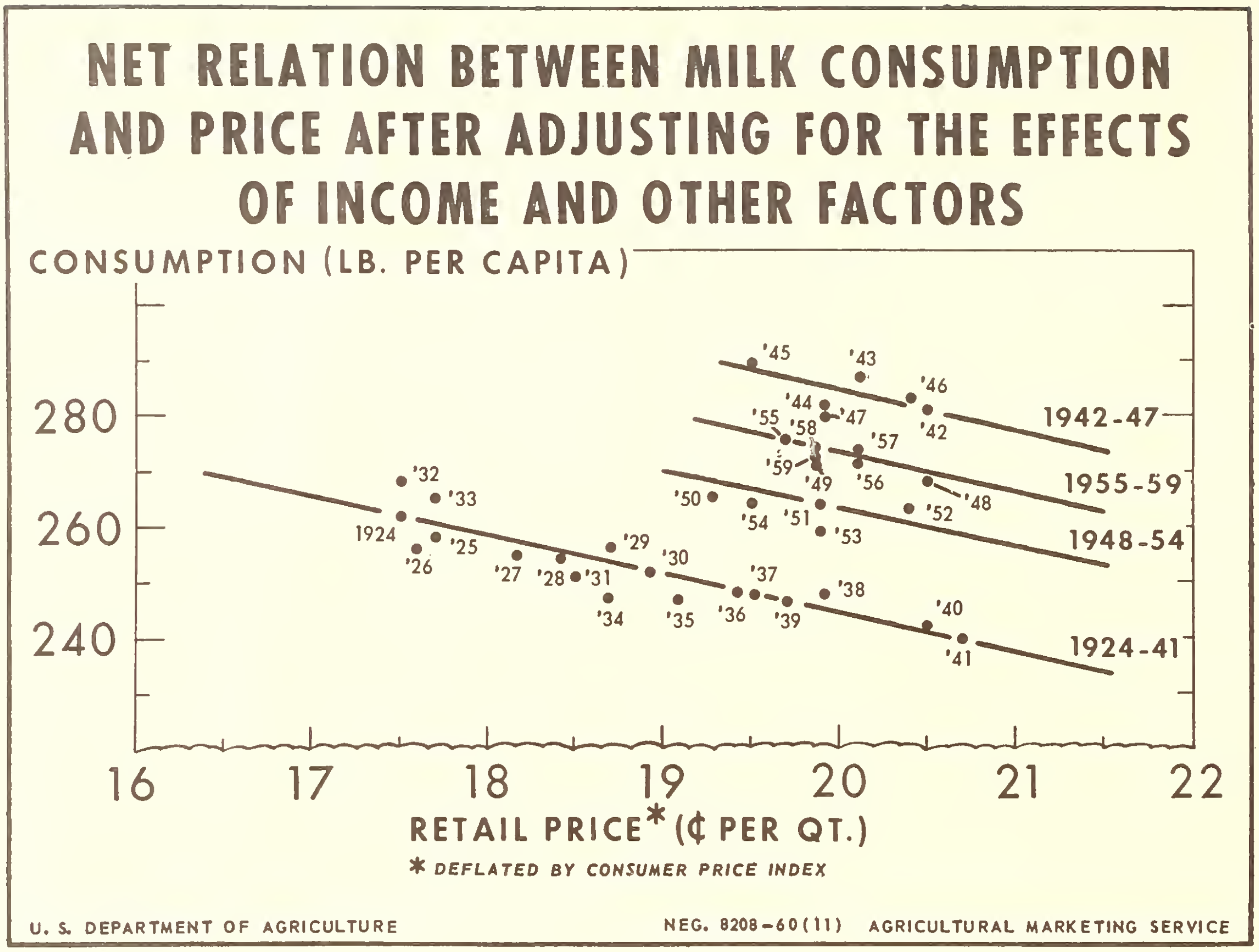

Figure 2.

more, carrying out a graphic analysis provides the analyst with insight about the data which he might otherwise miss.

Figures 1 to 5 indicate how graphic analysis can be used to provide insight as to the kind of regressions that should be run. Figure 2 indicates differences in levels of demand in the period between World War I and World War II, the period of World War II, and the two postwar periods. Demand analyses usually exclude the period of World War II. But by allowing for a shift in level, and by using discretionary income as an additional variable, the war years can be made part of the total analysis. Because of shortages in durable goods, consumers' cash position as reflected by discretionary income was strong; this position tended to increase expenditures for those items (fluid milk) that were available. The graph in figure 2 also appears to suggest that in 1948 and 1949, we were still adjusting from the high wartime levels even though actual consumption during these years was essentially at 1950–54 levels. Demand appeared to be relatively stable during 1950–54. The higher level for 1955–59, of course, was due to the introduction of the special milk program in 1954. The graphic analysis in this section provided the basis for equation (3). (See page 51.)

How Many Variations Should We Try?

Agricultural economists and statisticians frequently like to try several variations in conducting regression analyses. These variations can be

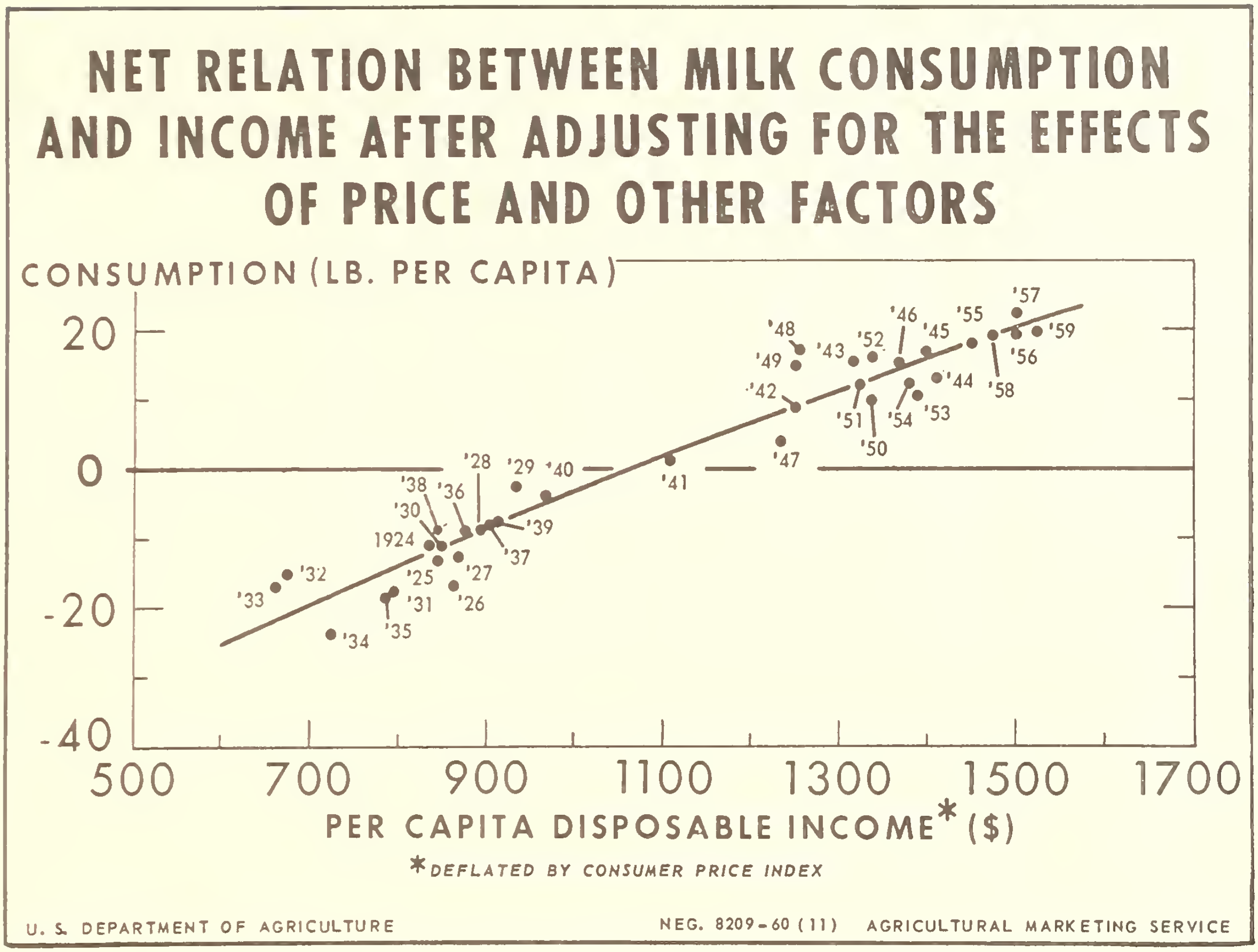

Figure 3.

grouped into three broad categories. The first concerns itself with basic changes in structure as indicated by the graphic analysis in the previous section of this paper. The second deals with the form of relationship—linear, curvilinear, and so on. The third is concerned with refinement of the data including use of series that may have a different conceptual base.

Let us examine the effect that each type of variation has on the expected results from regression analysis. There is no question but that the analyst will improve his results if he allows properly for basic changes in structure. The comparison of the first four demand regressions attests to this fact. (See pages 51 and 52.) This is further verified by the quarterly analyses for fluid whole milk. (See page 44.)

Use of the proper form of relationship can also be important. But unless there is a substantial difference, results will tend to be similar for several variations of this type. Review of past analyses of demand for fluid milk and dairy products suggests that the form of relationship is not too important in most instances.

Should we try variations that are really refinements in data, or different variations of the same basic set of data? The purpose of showing the many demand regressions on pages 51 and 52 is to illustrate the point that refinements in the third group may have little effect on the interpretation of the analysis. For this reason, if one obtains poor results in the first analysis, one should not expect improvements from refinements of this kind.

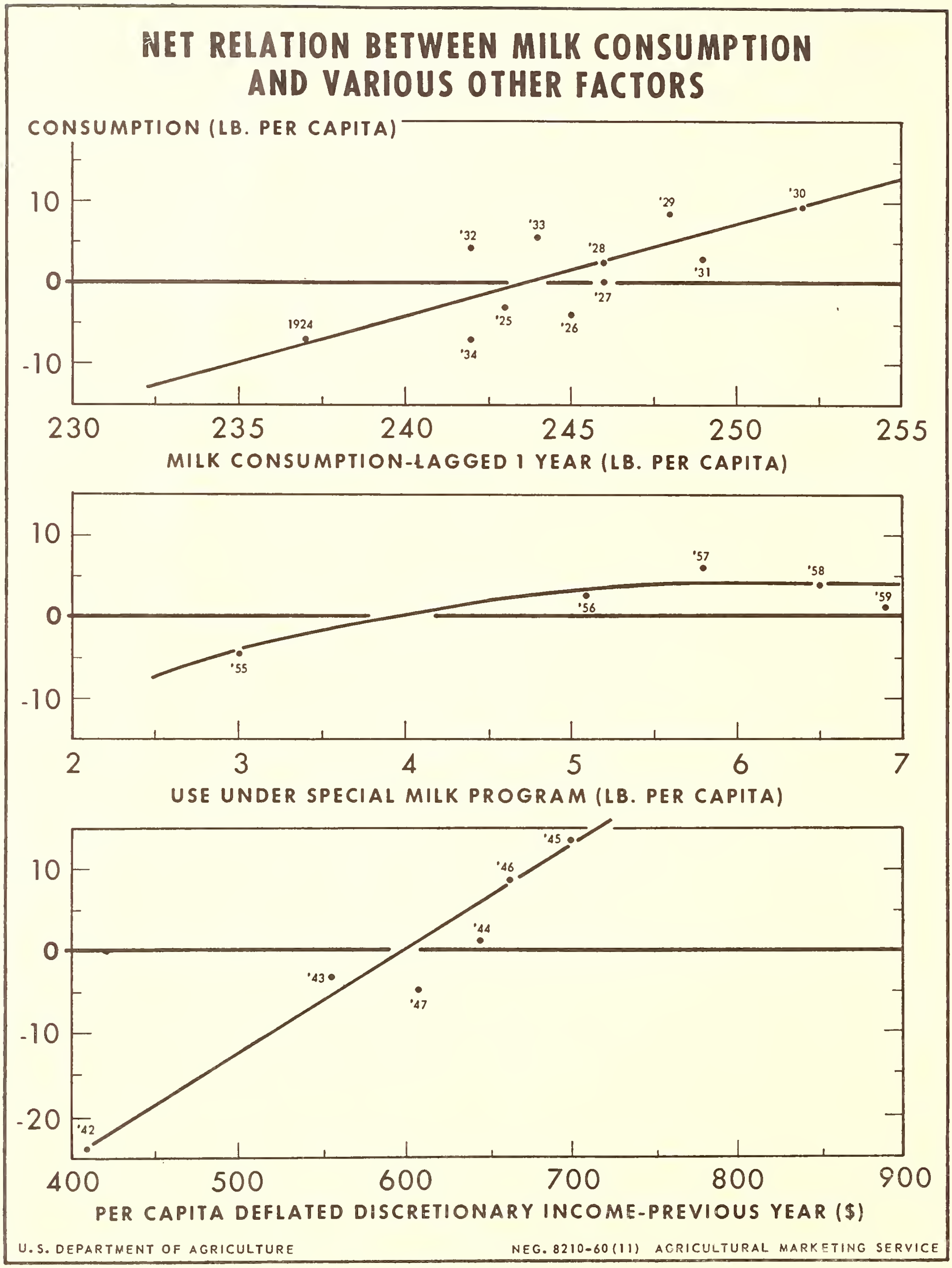
NET RELATION BETWEEN MILK CONSUMPTION AND VARIOUS OTHER FACTORS
CONSUMPTION (LB. PER CAPITA)
10
0
-10
1924
'25
'26
'27
'28
'29
'30
'31
'32
'33
'34
230
235
240
245
250
255
MILK CONSUMPTION-LAGGED 1 YEAR (LB. PER CAPITA)
'55
'56
'57
'58
'59
2
3
4
5
6
7
USE UNDER SPECIAL MILK PROGRAM (LB. PER CAPITA)
-20
'42
'43
'44
'45
'46
'47
400
500
600
700
800
900
PER CAPITA DEFLATED DISCRETIONARY INCOME-PREVIOUS YEAR ($)
U.S. DEPARTMENT OF AGRICULTURE
NEG. 8210-60 (11) AGRICULTURAL MARKETING SERVICE

Figure 4.

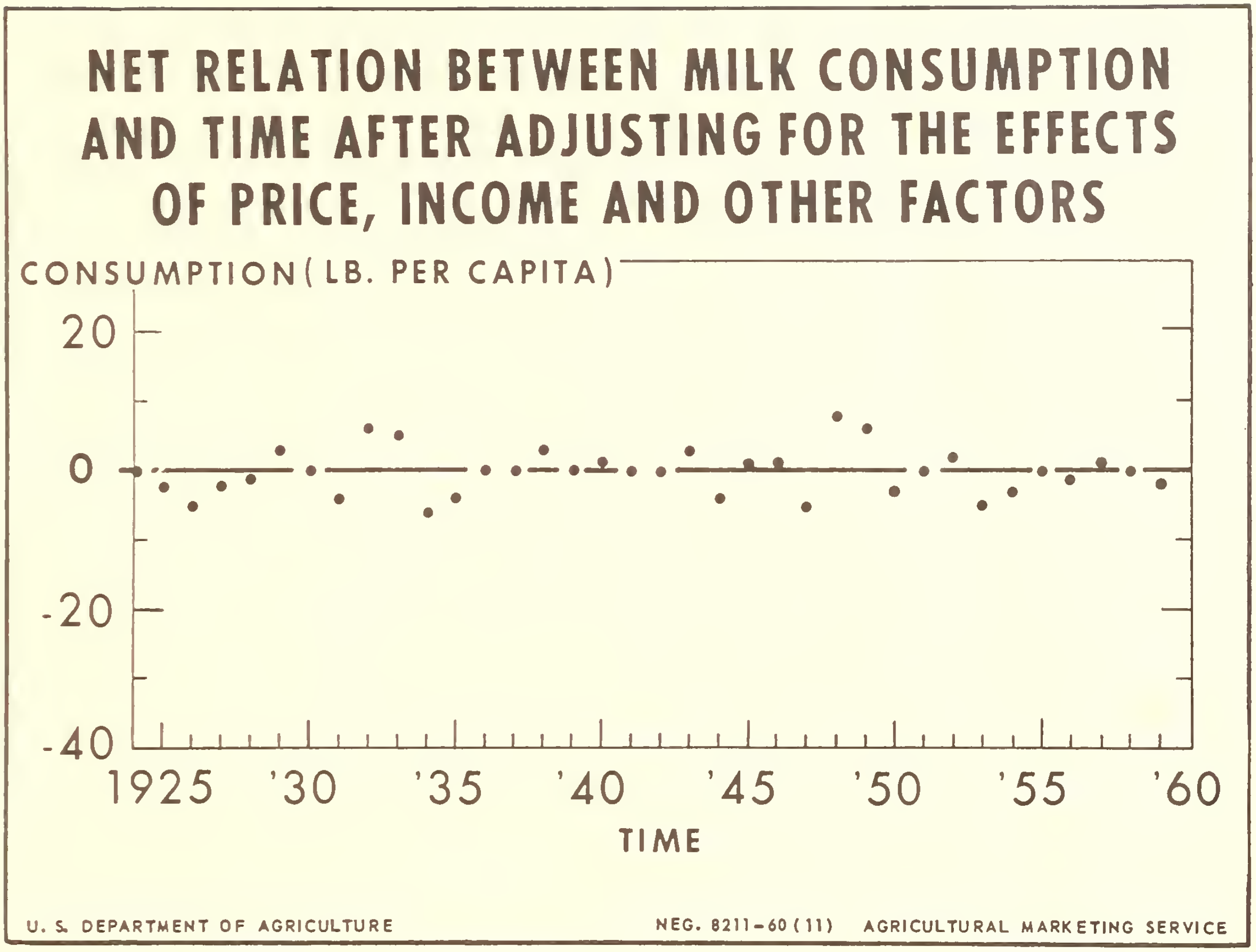

Figure 5.

Regressions for Demand for Fluid Milk [13]

Based on data in logarithms for 1924–59:

(1) $X_1' = 3.2022 - \underset{\substack{(.17)\\(.27)}}{.62X_2} + \underset{\substack{(.03)\\(.83)}}{.41X_3}$

$R^2_{1.23} = .87 \qquad s_{1.23} = .02$

(2) $X_1' = 1.5087 - \underset{\substack{(.12)\\(.17)}}{.32X_2} + \underset{\substack{(.04)\\(.50)}}{.21X_3} + \underset{\substack{(.08)\\(.59)}}{.54X_4}$

$R^2_{1.234} = .95 \qquad s_{1.234} = .01$

(3) $X_1' = 2.9658 - \underset{\substack{(.12)\\(.33)}}{.44X_2} + \underset{\substack{(.04)\\(.50)}}{.23X_3} + \underset{\substack{(.008)\\(.62)}}{.058X_5} + \underset{\substack{(.008)\\(.27)}}{.026X_6} + \underset{\substack{(.010)\\(.35)}}{.040X_7}$

$R^2_{1.23567} = .96 \qquad s_{1.23567} = .009$

---

[13] The variables are identified at the end of the formulas. Numbers in upper parentheses are standard errors of the regression coefficients; lower parentheses contain partial coefficients of determination.

(4) $$\begin{array}{lllllll} X_1'=2.0731- & .37X_2+ & .22X_3+ & 34X_4+ & .029X_5+ & .005X_6+ & .013X_7 \\ & (.10) & (.04) & (.10) & (.012) & (.009) & (.012) \\ & (.31) & (.55) & (.26) & (.17) & (.009) & (.04) \end{array}$$
$R^2_{1.234567}=.97$ $\quad s_{1.234567}=.008$

(5) $$\begin{array}{lllllll} X_1'=3.2010- & .64X_2+ & .26X_3+ & .051X_5+ & .017X_6+ & .029X_7+ & .014X_8 \\ & (.19) & (.05) & (.010) & (.010) & (.013) & (.011) \\ & (.27) & (.50) & (.48) & (.09) & (.14) & (.05) \end{array}$$
$R^2_{1.235678}=.96$ $\quad s_{1.235678}=.009$

(6) $$\begin{array}{lllllll} X_1'=2.9039- & .39X_2+ & .10X_3+ & .046X_5+ & .015X_6+ & .028X_7+ & .18X_9 \\ & (.11) & (.06) & (.009) & (.008) & (.010) & (.07) \\ & (.32) & (.08) & (.49) & (.11) & (.21) & (.21) \end{array}$$
$R^2_{1.235679}=.97$ $\quad s_{1.235679}=.009$

(7) $$\begin{array}{llllll} X_1'=2.5814+ & .14X_3+ & .063X_5+ & .026X_6+ & .048X_7- & .14X_{10} \\ & (.04) & (.009) & (.010) & (.011) & (.07) \\ & (.30) & (.59) & (.19) & (.38) & (.11) \end{array}$$
$R^2_{1.356710}=.94$ $\quad s_{1.356710}=.011$

(8) $$\begin{array}{lllllll} X_1'=1.6156+ & .14X_3+ & .37X_4+ & .030X_5+ & .005X_6+ & .018X_7- & .083X_{10} \\ & (.03) & (.13) & (.014) & (.011) & (.014) & (.067) \\ & (.36) & (.23) & (.14) & (.007) & (.05) & (.05) \end{array}$$
$R^2_{1.3456710}=.96$ $\quad s_{1.3456710}=.010$

Description of Variables:

X_1—Fluid whole milk, civilian nonfarm consumption, pounds per person.
X_2—Retail price for fluid milk (AMS series) deflated by CPI, cents per quart.
X_3—Disposable income deflated by CPI, dollars per person.
X_4—X_1 lagged one year.
X_5—Value of 1 for all years except 1943–47 when the value of 10 is used.
X_6—Value of 1 for all years except 1948–54 when value of 10 is used.
X_7—Value of 1 for all years except 1955–59 when value of 10 is used.
X_8—Time trend, 1924=1.
X_9—X_3 lagged one year.
X_{10}—Retail price for fluid milk (AMS series) deflated by CPI, food, cents per quart.

Literature Cited

(*1*) Brandow, G. E.
1958. A NOTE ON THE NERLOVE ESTIMATE OF SUPPLY ELASTICITY. Jour. Farm Econ. 40:719–722.

(*2*) Breimyer, Harold F.
1955. ELMER WORKING: THE DEMAND FOR MEAT. Agr. Econ. Research. 7:73–77.

(*3*) Buse, Rueben C.
1958. TOTAL ELASTICITIES—A PREDICTIVE DEVICE. Jour. Farm Econ. 40:881–891.

(*4*) Christ, Carl F.
1960. SIMULTANEOUS EQUATION ESTIMATION: ANY VERDICT YET? Econometrica. 28:835–845.

(*5*) Cochrane, D., and Orcutt, G. H.
1949. APPLICATION OF LEAST SQUARES REGRESSION TO RELATIONSHIPS CONTAINING AUTOCORRELATED ERROR TERMS. Jour. Amer. Statis. Asso. 44:32–61, illus.

(*6*) Foote, Richard J.
1955. A COMPARISON OF SINGLE AND SIMULTANEOUS EQUATION TECHNIQUES. Jour. Farm Econ. 37:975–990.

(*7*) ———
1958. ANALYTICAL TOOLS FOR STUDYING DEMAND AND PRICE STRUCTURES U.S. Dept. Agr. Agr. Handb. 146.

(*8*) ——— and Waugh, Frederick V.
1957. RESULTS OF AN EXPERIMENT TO TEST THE FORECASTING MERITS OF LEAST SQUARES AND LIMITED INFORMATION EQUATIONS. U.S. Agr. Marketing Serv.

(*9*) ——— and Weingarten, Hyman

1956. HOW RESEARCH RESULTS CAN BE USED TO ANALYZE ALTERNATIVE GOVERNMENTAL POLICIES. Agr. Econ. Research. 8:33–43.

(*10*) Fox, Karl A., and Cooney, James F.
1954. EFFECTS OF INTERCORRELATION UPON MULTIPLE CORRELATION AND REGRESSION MEASURES. U.S. Agr. Marketing Serv. AMS–341.

(*11*) Gerra, Martin J.
1959. THE DEMAND, SUPPLY, AND PRICE STRUCTURE FOR EGGS. U.S. Dept. Agr. Tech. Bull. 1204.

(*12*) Gislason, Conrad
1957. A NOTE ON LONG-RUN PRICE ELASTICITY. Jour. Farm Econ. 39:798–802.

(*13*) Haavelmo, Trygve
1943. THE STATISTICAL IMPLICATIONS OF A SET OF SIMULTANEOUS EQUATIONS. Econometrica. 11:1–12.

(*14*) Hildreth, Clifford
1960. SIMULTANEOUS EQUATIONS: ANY VERDICT YET? Econometrica. 28:846–854.

(*15*) Klein, L. R.
1960. SINGLE EQUATION VS. EQUATION SYSTEM METHODS OF ESTIMATION IN ECONOMETRICS. Econometrica. 28:866–871.

(*16*) Koopmans, Tjalling C.
1953. IDENTIFICATION PROBLEMS IN ECONOMIC MODEL CONSTRUCTION. In Studies in Econometric Method. Cowles Commission for Research in Economics Monogr. 14.

(*17*) Kuznets, G. M.
1953. MEASUREMENT OF MARKET DEMAND WITH PARTICULAR REFERENCE TO CONSUMER DEMAND FOR FOOD. Jour. Farm Econ. 35:878–895.

(*18*) Ladd, G. W.
1960. A STATISTICAL ANALYSIS OF CERTAIN INSTITUTIONAL VARIABLES IN THE BUTTER AND MARGARINE MARKET. Iowa Agr. Expt. Sta. Research Bull. 474.

(*19*) Ladd, George W., and Tedford, John R.
1959. A GENERALIZATION OF THE WORKING METHOD FOR ESTIMATING LONG-RUN ELASTICITIES. Jour. Farm Econ. 41:221–233.

(*20*) Liu, Ta-Chung
1960. UNDERIDENTIFICATION, STRUCTURAL ESTIMATION, AND FORECASTING. Econometrica. 28:855–865.

(*21*) MacPherson, D. D., and Smith, Helen V.
1959. MILK DISTRIBUTORS' SALES AND COSTS, JULY-SEPTEMBER 1959. U.S. Dept. Agr. MDSC–12.

(*22*) Meinken, Kenneth W.
1955. THE DEMAND AND PRICE STRUCTURE FOR WHEAT. U.S. Dept. Agr. Tech. Bull. 1136.

(*23*) ———, Rojko, Anthony S., and King, Gordon A.
1956. MEASUREMENT OF SUBSTITUTION IN DEMAND FROM TIME SERIES DATA—A SYNTHESIS OF THREE APPROACHES. Jour. Farm Econ. 38: 711–735.

(*24*) Mighell, R. L., and Allen, R. H.
1939. DEMAND SCHEDULES—"NORMAL" AND "INSTANTANEOUS" Jour. Farm Econ. 21: 555–569.

(*25*) Nerlove, Marc
1958. DISTRIBUTED LAGS AND THE ESTIMATION OF LONG-RUN SUPPLY AND DEMAND ELASTICITIES: THEORETICAL CONSIDERATIONS. Jour. Farm Econ. 40: 301–311.

(*26*) ———
1958. DISTRIBUTED LAGS AND DEMAND ANALYSIS FOR AGRICULTURAL COMMODITIES. U.S. Dept. Agr. Agr. Handb. 141.

(*27*) ——— and Addison, William
1958. STATISTICAL ESTIMATION OF LONG-RUN ELASTICITIES OF SUPPLY AND DEMAND. Jour. Farm Econ. 40: 861–880.

(*28*) ——— and Waugh, Frederick V.
ADVERTISING WITHOUT SUPPLY CONTROL: SOME IMPLICATIONS OF A STUDY OF THE ADVERTISING OF ORANGES. Submitted to Jour. Farm Econ.

(*29*) O'Regan, William G.
1955. DEMAND FOR MEAT. Jour. Farm Econ. 37: 752–755.

(*30*) Rojko, Anthony S.
1957. ECONOMETRIC MODELS FOR THE DAIRY INDUSTRY. Jour. Farm Econ. 39: 323–338.

(*31*) ———
1957. THE DEMAND AND PRICE STRUCTURE

FOR DAIRY PRODUCTS. U.S. Dept. Agr. Tech. Bull. 1168.

(32) TINTNER, GERHARD
1952. ECONOMETRICS. New York. 370 pp.

(33) WAUGH, FREDERICK V.
1956. A PARTIAL INDIFFERENCE SURFACE FOR BEEF AND PORK. Jour. Farm Econ. 38: 102–112.

(34) ———
1957. GRAPHIC ANALYSIS IN AGRICULTURAL ECONOMICS. U.S. Dept. Agr. Agr. Handb. 128.

(35) ———
1959. NEEDED RESEARCH ON THE EFFECTIVENESS OF FARM PRODUCTS PROMOTIONS. Jour. Farm Econ. 41: 364–376.

(36) ———
THE PLACE OF LEAST SQUARES IN ECONOMETRICS. To be published in Econometrica, July 1961.

(37) WOLD, HERMAN O. A.
1959. ENDS AND MEANS IN ECONOMETRIC MODEL BUILDING. In Grenander, Ulf, ed., Probability and Statistics: The Harald Cramér Volume, 434 pp., illus. New York.

(38) WOLD, H., AND FAXER, P.
1957. ON THE SPECIFICATION ERROR IN REGRESSION ANALYSIS. Annals Math. Statis. 28: 265–267.

(39) WORKING, ELMER
1927. WHAT DO STATISTICAL "DEMAND CURVES" SHOW? Quart. Jour. Econ. 4: 212–235, illus.

(40) WORKING, ELMER J.
1954. DEMAND FOR MEAT. Chicago: Institute of Meat Packing.